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RESEARCH REPORT No. EM-145

A New Method for the Determination of Far Fields With Applications to the Problem of Radiation of a Line Source at the Tip of an Absorbing Wedge

F. C. KARAL, JR. and S. N. KARP

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Abstract

A new simple operational method for the determination of the diffracted far field for wedges with impedance boundary conditions is presented. The exterior angle of wedge is $2\pi - \frac{\pi}{2n}$, where n is an integer. The excitation may be either an incident plane wave or a line source. The method is described in detail for the case of an absorbing wedge with a line source located at the tip. Several examples are given to illustrate the method. It is also shown that the method can be applied directly to the classical problem of perfectly conducting wedges.

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I. Introduction

In this paper we present a simple method for the determination of the diffracted far field for wedges with impedance boundary conditions. The excitation may be either an incident plane wave or a line source located at the tip of the wedge. The latter case is emphasized. The impedance boundary condition considered here is valid for the case of an absorbing surface. The method to be described, however, need not be restricted to absorbing surfaces but also applies to surfaces that support surface waves. Several examples are given to illustrate the method. It is also shown that the simple method developed here can be applied directly to the classical problem of diffraction by a perfectly conducting wedge or half plane.

The impedance boundary conditions prescribed on the wedge surfaces are given by

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \lambda u = 0 \qquad \theta = 0 \tag{1-1}$$

$$\frac{\partial u}{\partial \theta} = 0 \qquad \theta = \alpha \tag{1-2}$$

where u is the z-component of the magnetic vector, r and θ are the usual polar coordinates, α is the angular space for which the solution is desired and λ is a constant characteristic of the wedge surface.

When the surface is absorbing the value of λ is given by

$$\lambda = iks \tag{1-3}$$

where k is the free space propagation constant and s is a real positive parameter characteristic of the absorbing material. By an absorbing surface we mean a plane surface or screen that absorbs a maximum amount of radiation for certain angles of incidence of an incoming plane wave. If the parameter is restricted to lie between 0 and 1, the screen absorbs all radiation for some particular direction of incidence and partially absorbs radiation for other angles of incidence. Hence the absorbing surface under discussion here can act like a black screen for certain directions of incidence. For additional information on absorbing surfaces see[2], and for information on diffraction by black screens see Baker and Copson[1].

The problems treated here are not separable because of the mixed boundary conditions. It is possible, however, to introduce an operator L which defines an auxiliary function $Lu \equiv v$, such that the auxiliary function satisfies the wave equation and simple homogeneous boundary conditions on both wedge surfaces. Once the auxiliary function is found, and this is not difficult, the original field could be determined by solving a partial differential equation. The boundary conditions in the original problem and certain necessary continuity conditions are then imposed which allow one to obtain an exact formal solution valid for the entire region of physical space. This basic idea is due to Stoker[9] and Lewy[8] and has been adapted by the authors to the solution of problems in diffraction theory. See[2]-[6] . However, if only the radiated far field is desired, it is possible to proceed in a much more simple and direct manner that avoids completely the detailed formal analysis which is quite lengthy and complicated. It is this simple and powerful method that is one of the main results of this paper. The other new results relate to the excitation of an absorbing wedge by a line source at the tip.

In Section II we determine the operator L that transforms the original impedance boundary conditions into simpler boundary conditions involving an auxiliary function whose solution is known. In Section III we make use of this operator and examine the problem of an incident plane wave striking a wedge of angle $\frac{\pi}{2n}$ (exterior angle $2\pi - \frac{\pi}{2n}$), where n is an integer, when an impedance boundary condition is prescribed on the upper wedge surface and a normal derivative boundary condition is prescribed on the lower wedge surface. The radiated far field is determined as a quotient of two trigonometric expressions. The numerator contains a set of unknown constants. The denominator has a corresponding number of zeros which would lead to undesirable shadow lines in the pattern. The constants in the numerator are then so determined as to eliminate these inadmissible poles or shadow lines.

The principal section is Section IV. In that section we determine the radiated far field of a line source located at the tip of a wedge of exterior angle $2\pi - \frac{\pi}{2n}$ when an impedance condition is prescribed on the upper face, and a Neumann condition is prescribed on the lower face.

Section V is devoted to illustrative examples of the general result of Section IV.

In Section VI it is shown how the method we have described can be employed in the direct derivation of the diffraction pattern for perfectly conducting wedges. In order to illustrate a certain alternative which is available in the application of the preceding method we determine various constants in the far field amplitude for diffraction by a conducting half plane and for a right angled wedge by the use of the reciprocity theorem rather than by the method of regularity (i.e., avoidance of undesirable poles or shadow lines) which we exploited in the preceding sections.

II. Determination of Operator

In the problems that follow we shall need an operator L that transforms the impedance boundary condition given by

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \lambda u = 0 \qquad \theta = 0 \qquad (2\text{-}1)$$

$$\frac{\partial u}{\partial \theta} = 0 \qquad \theta = \alpha \qquad (2\text{-}2)$$

into the simpler boundary conditions given by

$$v = 0 \qquad \theta = 0 \qquad (2\text{-}3)$$

$$\frac{\partial v}{\partial \theta} = 0 \qquad \theta = \alpha \qquad (2\text{-}4)$$

From Stoker's paper on water waves [9], we are led immediately to the following operator

$$L = \prod_{p=1}^{n} \left(\alpha_{2p} D + \lambda\right) \qquad (2\text{-}5)$$

where

$$\alpha_k = e^{-i\frac{\pi}{2}\left(\frac{k}{n}+1\right)} \qquad k = 2, 4, \ldots\ldots 2n \qquad (2\text{-}6)$$

and D is the complex operator $\frac{d}{dz}$. Along the real axis D has the value $\frac{\partial}{\partial x}$, and along the imaginary axis the value $\frac{\partial}{\partial(iy)}$. Hence the appropriate operator for a wedge of angle $\frac{\pi}{2n}$ with an impedance boundary condition on the upper surface and a zero normal derivative condition on the lower surface becomes

$$L = -\prod_{p=1}^{n} \left\{ \sin\frac{\pi p}{n}\frac{\partial}{\partial x} + \cos\frac{\pi p}{n}\frac{\partial}{\partial y} - \lambda \right\} \qquad (2\text{-}7)$$

The transformation that simplifies the original boundary conditions (2-1) and (2-2) is

$$v = L u \qquad (2\text{-}8)$$

We make the important observation that the same operator and transformation applies to the case of a wedge of twice the exterior angle when the lower face has the same impedance boundary condition as the upper face. This fact follows immediately from symmetry.

III. Diffracted Far Field Due to an Incident Plane Wave

Consider a wedge of angle $\frac{\pi}{2n}$, where n is an integer, defined by the surfaces $\theta = 0$ and $\theta = \alpha$ as shown in Figure I. In the angular region $0 \leq \theta \leq \alpha$ we assume that we have free space. A plane wave u_I, whose magnetic vector is linearly polarized in the z-direction, is incident on the wedge. If the angle of incidence is θ_0 and the direction of the normal to the plane wave front is as shown in Figure 1, then

$$u_I = U_0 \, e^{-ik(x\cos\theta_0 + y\sin\theta_0)} \tag{3-1}$$

where U_0 is the magnitude of the incident plane wave and the time dependence $e^{-i\omega t}$ is omitted for convenience. We assume the boundary conditions on the wedge are given by

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \lambda u = 0 \qquad \theta = 0 \tag{3-2}$$

$$\frac{\partial u}{\partial \theta} = 0 \qquad \theta = \alpha \tag{3-3}$$

where u is the z-component of the magnetic vector and λ has already been defined. It is easily shown that in two dimensions the field component u is independent of z and determines completely the electromagnetic field. The field component $H_z \equiv u$ satisfies the wave equation

$$(\nabla^2 + k^2)u = 0 \tag{3-4}$$

where ∇^2 is the rectangular Laplacian and k is the propagation constant of free space. From (3-1), (2-7) and (2-8) we obtain the form of the incident plane wave in the transformed problem. We have

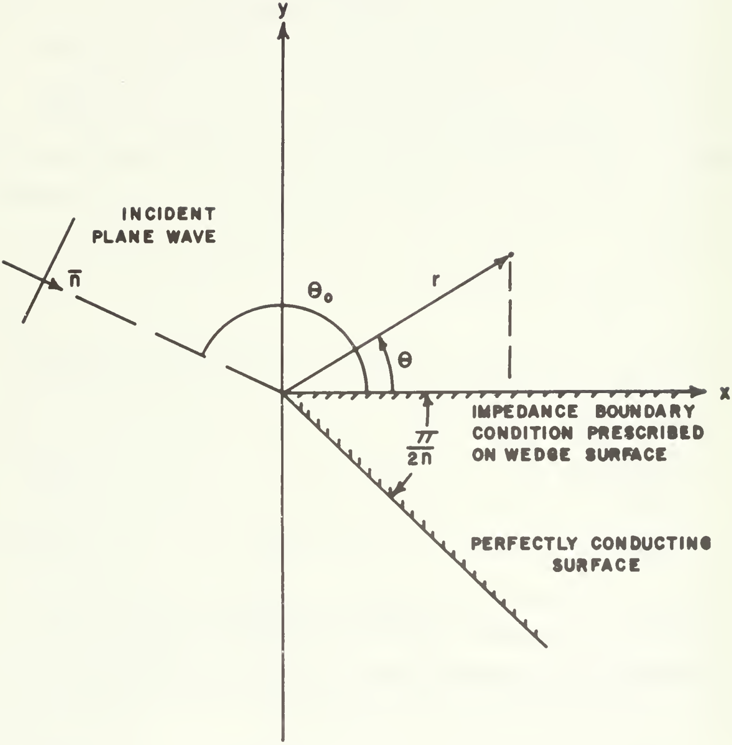
y
INCIDENT PLANE WAVE
$\bar{n}$
θ_0
r
θ
x
IMPEDANCE BOUNDARY CONDITION PRESCRIBED ON WEDGE SURFACE
$\frac{\pi}{2n}$
PERFECTLY CONDUCTING SURFACE

FIGURE 1

$$v_I = V_0\, e^{-ik(x\cos\theta_0 + y\sin\theta_0)} \tag{3-5}$$

where

$$V_0 = ik\, U_0 \prod_{p=1}^{n} \left\{ \sin\left(\frac{p\pi}{n} + \theta_0\right) + \sin\theta_1 \right\} \tag{3-6}$$

and

$$\lambda = ik\sin\theta_1 \tag{3-7}$$

The quantity u in the far field can be written as the sum of two terms. The first term can be found by using geometrical optics and is composed of incident and reflected waves. The second term is due to the diffracted field. We have

$$u = (u)_{geom} + (u)_{diff} \tag{3-8}$$

where $(u)_{geom}$ is assumed to be known and $(u)_{diff}$ is to be found. The transformed quantity v in the far field can be written in the form

$$v = (v)_{geom} + \left[(v)_{diff}\right]_F + \left[(v)_{diff}\right]_S \tag{3-9}$$

The diffracted far field solution of the transformed problem which is finite at the edge of the wedge and is denoted by the subscript F is assumed to be known and can be obtained by standard methods. The far field solution of the transformed problem which is singular at the edge of the wedge, and is denoted by the subscript S, is the asymptotic form of

$$\left[v\right]_S = \sum_{p=1}^{2n-1} c_p\, H^{(1)}_{\frac{(2p-1)n}{4n-1}}(kr) \sin\frac{(2p-1)n}{4n-1}\theta \tag{3-10}$$

The first term can be found by using geometrical optics and equations (2-7) and (2-8), and is assumed to be known. The far field expression for the finite solution is given by

$$[(v)_{diff}]_p \approx \sqrt{\frac{2}{\pi k r}}\, e^{ikr - i\frac{\pi}{4}}\, \tilde{B}(\theta,\theta_0) \tag{3-11}$$

where

$$\tilde{B}(\theta,\theta_0) = k\,\mathcal{U}_0 \prod_{p=1}^{n} \left\{\sin\left(\tfrac{p\pi}{n}+\theta_0\right) + \sin\theta_1\right\} \tilde{\tilde{B}}(\theta,\theta_0) \tag{3-12}$$

and

$$\tilde{\tilde{B}}(\theta,\theta_0) = \frac{\frac{n}{4n-1}\sin\frac{n\theta_0}{4n-1}\sin\frac{n\pi}{4n-1}\sin\frac{n\theta}{4n-1}}{\left[\cos\frac{n(\theta-\theta_0)}{4n-1} - \cos\frac{n\pi}{4n-1}\right]\left[\cos\frac{n(\theta+\theta_0)}{4n-1} - \cos\frac{n\pi}{4n-1}\right]} \tag{3-13}$$

The far field expression for the singular solution is given by

$$[(v)_{diff}]_S \approx \sqrt{\frac{2}{\pi k r}}\, e^{ikr - i\frac{\pi}{4}} \sum_{p=1}^{2n-1} \tilde{c}_{pn} \sin\frac{n(2p-1)}{4n-1}\theta \tag{3-14}$$

where

$$\tilde{c}_{pn} = c_p\, e^{-i\left[\frac{n(2p-1)}{4n-1}\right]\frac{\pi}{2}} \tag{3-15}$$

We expect a priori that the far field form of the original field u is of the form

$$(u)_{diff} \approx \sqrt{\frac{2}{\pi k r}}\, e^{ikr - i\frac{\pi}{4}}\, \tilde{n}(\theta) \tag{3-16}$$

where $\tilde{n}(\theta)$ is unknown. Consider now only the diffracted part of the far field expressions. We have

$$(v)_{diff} = L\left\{(u)_{diff}\right\} \tag{3-17}$$

or

$$\left[(v)_{diff}\right]_F + \left[(v)_{diff}\right]_S = -\prod_{p=1}^{n}\left\{\sin\frac{\pi p}{n}\frac{\partial}{\partial x} + \cos\frac{\pi p}{n}\frac{\partial}{\partial y} - \lambda\right\}(u)_{diff} \tag{3-18}$$

But in the far field

$$\frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{1}{r}\sin\theta\frac{\partial}{\partial\theta} \approx \cos\theta\frac{\partial}{\partial r} \approx ik\cos\theta \tag{3-19}$$

$$\frac{\partial}{\partial y} = \sin\theta\frac{\partial}{\partial r} + \frac{1}{r}\cos\theta\frac{\partial}{\partial\theta} \approx \sin\theta\frac{\partial}{\partial r} \approx ik\sin\theta \tag{3-20}$$

and hence

$$\left[(v)_{diff}\right]_F + \left[(v)_{diff}\right]_S = -\prod_{p=1}^{n}\left\{\sin\left(\frac{\pi p}{n}+\theta\right)\frac{\partial}{\partial r} - \lambda\right\}(u)_{diff} \tag{3-21}$$

If we substitute (3-7), (3-11), (3-14) and (3-16) into the above we obtain

$$\tilde{n}(\theta) = \frac{\left(\frac{i}{k}\right)\left[\tilde{B}(\theta,\theta_0) + \sum_{p=1}^{2n-1}\tilde{c}_{pn}\sin\frac{n(2p-1)}{4n-1}\theta\right]}{\prod_{p=1}^{n}\left[\sin\left(\frac{\pi p}{n}+\theta\right) - \sin\theta_1\right]} \tag{3-22}$$

The constants $\tilde{c}_{pn}$ are as yet unknown.

Notice, however, that the denominator of (3-22) vanishes for certain values of θ. Let the roots of the equation

$$\prod_{p=1}^{n}\left[\sin\left(\frac{\pi p}{n}+\theta\right) - \sin\theta_1\right] = 0 \tag{3-23}$$

be denoted by β_j where j is an integer. There are 2n-1 values of j, (i.e., there are 2n-1 roots of (3-23) in the interval $0 < \theta < 2\pi - \frac{\pi}{2n}$ and at these roots there must be no poles of $\tilde{n}(\theta)$. The reason that the far field amplitude $\tilde{n}(\theta)$ can have no infinities at the roots of (3-23) is that these would contribute inadmissible shadow lines. Hence the numerator of (3-22) must vanish at the roots β_j of the denominator. Therefore we can write

$$(u)_{diff} = \sqrt{\frac{2}{\pi k r}}\, e^{i\left(kr+\frac{\pi}{4}\right)}\,\mathcal{U}_0\,\frac{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}+\theta_0\right)+\sin\theta_1\right\}\; N_{2n-1}}{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}-\theta\right)-\sin\theta_1\right\}\; \Delta_{2n-1}} \tag{3-24}$$

where

$$N_{2n-1}=\begin{vmatrix} \tilde{\tilde{B}}(\theta,\theta_0) & \sin\frac{\pi}{4n-1}\theta & \sin 3\frac{\pi}{4n-1}\theta & \cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\theta \\ \tilde{\tilde{B}}(\beta_1,\theta_0) & \sin\frac{\pi}{4n-1}\beta_1 & \sin 3\frac{\pi}{4n-1}\beta_1 & \cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_1 \\ \tilde{\tilde{B}}(\beta_2,\theta_0) & \sin\frac{\pi}{4n-1}\beta_2 & \sin 3\frac{\pi}{4n-1}\beta_2 & \cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_2 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{\tilde{B}}(\beta_{2n-1},\theta_0) & \sin\frac{\pi}{4n-1}\beta_{2n-1} & \sin 3\frac{\pi}{4n-1}\beta_{2n-1} & \cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_{2n-1} \end{vmatrix} \qquad (3\text{-}25)$$

$$\Delta_{2n-1}=\begin{vmatrix} \sin\frac{\pi}{4n-1}\beta_1 & \sin 3\frac{\pi}{4n-1}\beta_1 & \cdots\cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_1 \\ \sin\frac{\pi}{4n-1}\beta_2 & \sin 3\frac{\pi}{4n-1}\beta_2 & \cdots\cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_2 \\ \vdots & \vdots & & \vdots \\ \sin\frac{\pi}{4n-1}\beta_{2n-1} & \sin 3\frac{\pi}{4n-1}\beta_{2n-1} & \cdots\cdots\cdots & \sin(4n-3)\frac{\pi}{4n-1}\beta_{2n-1} \end{vmatrix} \qquad (3\text{-}26)$$

The roots β_j are here regarded as being enumerated in some arbitrary but fixed order. We first consider the determinant Δ_{2n-1}. It can be shown that its value is given by

$$\begin{aligned} \Delta_{2n-1} = (2)^{2(2n-1)(n-1)} & \sin x_1 \sin x_2 \sin x_3 \cdots\cdots \sin x_{2n-1} \\ \cdot & [\cos^2 x_2-\cos^2 x_1][\cos^2 x_3-\cos^2 x_1][\cos^2 x_4-\cos^2 x_1]\cdots\cdots[\cos^2 x_{2n-1}-\cos^2 x_1] \\ & [\cos^2 x_3-\cos^2 x_2][\cos^2 x_4-\cos^2 x_2]\cdots\cdots[\cos^2 x_{2n-1}-\cos^2 x_2] \\ & [\cos^2 x_4-\cos^2 x_3]\cdots\cdots[\cos^2 x_{2n-1}-\cos^2 x_3] \\ & \ddots \qquad \vdots \\ & [\cos^2 x_{2n-1}-\cos^2 x_{2n-2}] \end{aligned} \qquad (3\text{-}27)$$

where

$$x_j = \frac{n}{4n-1}\beta_j \tag{3-28}$$

A principle minor of the numerator is of the form

$$(-1)^j \tilde{\tilde{B}}(\beta_j,\theta_0) \begin{vmatrix} \sin\phi & \sin 3\phi & \cdots\cdots & \sin(4n-3)\phi \\ \sin x_1 & \sin 3x_1 & & \sin(4n-3)x_1 \\ \sin x_2 & \sin 3x_2 & & \sin(4n-3)x_2 \\ \vdots & \vdots & & \vdots \\ \sin x_{j-1} & \sin 3x_{j-1} & & \sin(4n-3)x_{j-1} \\ \sin x_{j+1} & \sin 3x_{j+1} & & \sin(4n-3)x_{j+1} \\ \vdots & \vdots & & \vdots \\ \sin x_{2n-1} & \sin 3x_{2n-1} & \cdots\cdots & \sin(4n-3)x_{2n-1} \end{vmatrix} \tag{3-29}$$

where

$$\phi = \frac{n}{4n-1}\theta \tag{3-30}$$

Note that the $j^{\underline{th}}$ row is omitted. Expanding the above as before we have

$$\begin{aligned} &(-1)^j \tilde{\tilde{B}}(\beta_j,\theta_0)(2)^{2(2n-1)(n-1)} \sin\phi \sin x_1 \sin x_2 \cdots \sin x_{j-1} \sin x_{j+1} \cdots\cdots \sin x_{2n-1} \\ &\cdot\left[\cos^2 x_1 - \cos^2\phi\right]\left[\cos^2 x_2 - \cos^2\phi\right]\cdots\cdots\left[\cos^2 x_{j-1} - \cos^2\phi\right]\left[\cos^2 x_{j+1} - \cos^2\phi\right]\cdots\cdots\left[\cos^2 x_{2n-1} - \cos^2\phi\right] \\ &\qquad\cdot\left[\cos^2 x_2 - \cos^2 x_1\right]\cdots\cdots\left[\cos^2 x_{j-1} - \cos^2 x_1\right]\left[\cos^2 x_{j+1} - \cos^2 x_1\right]\cdots\cdots\left[\cos^2 x_{2n-1} - \cos^2 x_1\right] \\ &\qquad\qquad\qquad\ddots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\vdots \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\left[\cos^2 x_{2n-1} - \cos^2 x_{2n-2}\right] \end{aligned} \tag{3-31}$$

Dividing numerator by denominator, and cancelling all common terms, we obtain

$$_{diff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})}\, U_0 \frac{n}{4n-1} \sin\frac{n\theta}{4n-1} \sin\frac{n\theta_0}{4n-1} \sin\frac{n\pi}{4n-1} \frac{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}+\theta\right)+\sin\theta_1\right\}}{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}+\theta\right)-\sin\theta_1\right\}}$$

$$\cdot\left[+\frac{1}{\left[\cos\frac{n(\theta-\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]\left[\cos\frac{n(\theta+\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]}\right.$$

$$-\frac{1}{\left[\cos\frac{n(\beta_1-\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]\left[\cos\frac{n(\beta_1+\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]}\frac{\left[\cos^2 x_2-\cos^2\phi\right]\left[\cos^2 x_3-\cos^2\phi\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2\phi\right]}{\left[\cos^2 x_2-\cos^2 x_1\right]\left[\cos^2 x_3-\cos^2 x_1\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2 x_1\right]}$$

$$-\frac{1}{\left[\cos\frac{n(\beta_2-\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]\left[\cos\frac{n(\beta_2+\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]}\frac{\left[\cos^2 x_1-\cos^2\phi\right]\left[\cos^2 x_3-\cos^2\phi\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2\phi\right]}{\left[\cos^2 x_1-\cos^2 x_2\right]\left[\cos^2 x_3-\cos^2 x_2\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2 x_2\right]}$$

$$-\frac{1}{\left[\cos\frac{n(\beta_3-\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]\left[\cos\frac{n(\beta_3+\theta_0)}{4n-1}-\cos\frac{n\pi}{4n-1}\right]}\frac{\left[\cos^2 x_1-\cos^2\phi\right]\left[\cos^2 x_2-\cos^2\phi\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2\phi\right]}{\left[\cos^2 x_1-\cos^2 x_3\right]\left[\cos^2 x_2-\cos^2 x_3\right]\cdots\cdots\left[\cos^2 x_{2n-1}-\cos^2 x_3\right]}$$

$$\left. - \;\cdots\cdots\cdots\cdots \right] \tag{3-32}$$

We can write (3-32) in a different form involving only sines. Further simplification yields the following compact expression

$$(u)_{diff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})}\, U_0 \frac{n}{4(4n-1)} \sin\frac{n\theta}{4n-1} \sin\frac{n\theta_0}{4n-1} \sin\frac{n\pi}{4n-1} \frac{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}+\theta_0\right)+\sin\theta_1\right\}}{\prod_{p=1}^{n}\left\{\sin\left(\frac{p\pi}{n}+\theta\right)-\sin\theta_1\right\}}$$

$$\cdot\left[+\frac{1}{\sin\frac{n(\theta-\theta_0+\pi)}{2(4n-1)}\sin\frac{n(\theta-\theta_0-\pi)}{2(4n-1)}\sin\frac{n(\theta+\theta_0+\pi)}{2(4n-1)}\sin\frac{n(\theta+\theta_0-\pi)}{2(4n-1)}}\,.\right.$$

$$-\sum_{j=1}^{2n-1}\frac{1}{\sin\frac{n(\beta_j-\theta_0+\pi)}{2(4n-1)}\sin\frac{n(\beta_j-\theta_0-\pi)}{2(4n-1)}\sin\frac{n(\beta_j+\theta_0+\pi)}{2(4n-1)}\sin\frac{n(\beta_j+\theta_0-\pi)}{2(4n-1)}} \tag{3-33}$$

$$\left.\cdot\frac{\prod_{\substack{p=1\\(p\neq j)}}^{2n-1}\sin\frac{n(\beta_p-\theta)}{4n-1}\sin\frac{n(\beta_p+\theta)}{4n-1}}{\prod_{\substack{p=1\\(p\neq j)}}^{2n-1}\sin\frac{n(\beta_p-\beta_j)}{4n-1}\sin\frac{n(\beta_p+\beta_j)}{4n-1}}\right]$$

This result is the diffracted far field due to an incident plane wave for a wedge of angle $\frac{\pi}{2n}$ (i.e. in an external angular space $2\pi - \frac{\pi}{2n}$) when an impedance boundary condition is prescribed on the upper face and a normal derivative boundary condition is prescribed on the lower. It is also the result for the diffracted far field for a wedge in an external space $4\pi - \frac{\pi}{n}$ ($0 < \theta < 4\pi - \frac{\pi}{n}$) when the same impedance boundary condition is prescribed on both surfaces, provided the incident wave arrives along the line of symmetry of the wedge.

IV. Diffracted Far Field Due to a Line Source Located at the Tip

Consider how a wedge of angle $\frac{\pi}{2n}$, where n is an integer, defined by the surfaçes $\theta = 0$ and $\theta = \alpha$ as shown in Figure 2. In the angular region $0 \leq \theta \leq \alpha$ we assume that we have free space. A magnetic line dipole is located at the tip of the wedge and has coordinates $x = 0$, $y = 0$. See Figure 2. We assume the boundary conditions on the wedge are given by

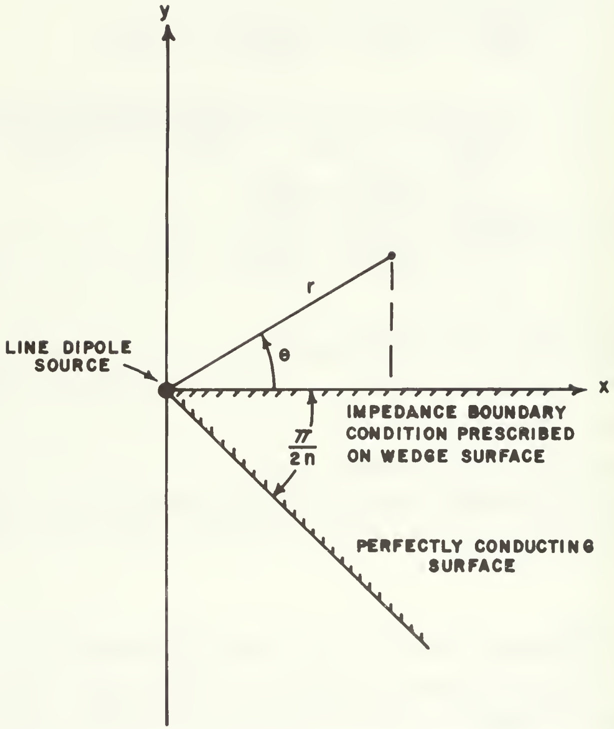
y
x
r
θ
LINE DIPOLE SOURCE
IMPEDANCE BOUNDARY CONDITION PRESCRIBED ON WEDGE SURFACE
$\frac{\pi}{2n}$
PERFECTLY CONDUCTING SURFACE

FIGURE 2

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \lambda u = 0 \qquad \theta = 0 \tag{4-1}$$

$$\frac{\partial u}{\partial \theta} = 0 \qquad \theta = \alpha \tag{4-2}$$

where u is the z-component of the magnetic vector and λ has already been defined. The field component $H_z \equiv u$ satisfies the wave equation

$$(\nabla^2 + k^2)u = -4\pi\delta(x-0)\delta(y-0) \tag{4-3}$$

where ∇^2 is the rectangular Laplacian, k is the propagation constant of free space and δ is the Dirac delta function.

The radiated far field is found in a manner similar to that used for the incident wave plane. Again we first introduce the operator

$$v = L u \tag{4-4}$$

where

$$L = -\prod_{p=1}^{n}\left\{\sin\frac{\pi p}{n}\frac{\partial}{\partial x} + \cos\frac{\pi p}{n}\frac{\partial}{\partial y} - \lambda\right\} \tag{4-5}$$

The inverse operation $u = L^{-1}v$ involves n integrations. Let v_s be that part of v which arises from performing integration on our source term. Therefore, v_s must be of the form

$$v_s = c_n H_n^{(1)}(kr)\sin n\theta \tag{4-6}$$

since the solution of (4-3) is given by the Green's function

$$u_s = i\pi H_0^{(1)}(kr) \tag{4-7}$$

In the above equations $H_n^{(1)}(kr)$ is a Hankel function of the first kind of order n and the subscript S indicates that these are source terms. The constant c_n is a normalization constant. The solution of the transformed problem is given by

$$v = c_n H_n^{(1)}(kr) \sin n\theta + \sum_{p=1}^{2n-1} c_p H^{(1)}_{\frac{(2p-1)n}{4n-1}}(kr) \sin \frac{(2p-1)n}{4n-1}\theta \tag{4-8}$$

and has the far field representation

$$\begin{aligned}(v)_{ff} \approx {} & c_n \sqrt{\frac{2}{\pi kr}}\, e^{i(kr-\frac{\pi}{4})} e^{-i\frac{n\pi}{2}} \sin n\theta \\ & + \sqrt{\frac{2}{\pi kr}}\, e^{i(kr-\frac{\pi}{4})} \sum_{p=1}^{2n-1} c_p e^{-i\frac{n(2p-1)}{4n-1}\frac{\pi}{2}} \sin \frac{n(2p-1)}{4n-1}\theta\end{aligned} \tag{4-9}$$

where v_{ff} denotes the far field form of v.

We expect a priori that the far field of the original field u is of the form

$$(u)_{ff} \approx \sqrt{\frac{2}{\pi kr}}\, e^{i(kr-\frac{\pi}{4})}\, \tilde{n}(\theta) \tag{4-10}$$

where $\tilde{n}(\theta)$ is unknown. If we now consider the far field of v we have

$$(v)_{ff} = -\prod_{p=1}^{n} \left\{ \sin\left(\frac{\pi p}{n}+\theta\right)\frac{\partial}{\partial r} - \lambda \right\} (u)_{ff} \tag{4-11}$$

Substituting from (4-10) and (4-11) we have

$$\begin{aligned}& c_n e^{-i\frac{n\pi}{2}} \sin n\theta + \sum_{p=1}^{2n-1} c_p e^{-i\frac{n(2p-1)}{4n-1}\frac{\pi}{2}} \sin \frac{n(2p-1)}{4n-1}\theta \\ & = -ik \prod_{p=1}^{n} \left\{ \sin\left(\frac{\pi p}{n}+\theta\right) - \sin\theta_1 \right\} \tilde{n}(\theta)\end{aligned} \tag{4-12}$$

where we have set

$$\lambda = ik\sin\theta_1 \tag{4-13}$$

Hence

$$(u)_{ff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})}\left(\frac{1}{k}\right)\frac{\left[\tilde{c}_n \sin n\theta + \sum_{p=1}^{2n-1} \tilde{c}_{pn} \sin \frac{n(2p-1)}{4n-1}\theta\right]}{\prod_{p=1}^{n}\left[\sin\left(\frac{\pi p}{n}+\theta\right)-\sin\theta_1\right]} \tag{4-14}$$

where

$$\tilde{c}_n = c_n e^{-i\frac{n\pi}{2}} \tag{4-15}$$

$$\tilde{c}_{pn} = c_p e^{-i\frac{n(2p-1)}{4n-1}\frac{\pi}{2}} \tag{4-16}$$

Using the same argument as in Section III, we note that $(u)_{ff}$ can be written in the form

$$(u)_{ff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})}\frac{\left(\frac{\tilde{c}_n}{k}\right)}{\prod_{p=1}^{n}\left[\sin\left(\frac{\pi p}{n}+\theta\right)-\sin\theta_1\right]}\,\frac{N_{2n-1}}{\Delta_{2n-1}} \tag{4-17}$$

where

$$N_{2n-1} = \begin{vmatrix} \sin n\theta & \sin\frac{n\theta}{4n-1} & \sin 3\frac{n\theta}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\theta}{4n-1} \\ \sin n\beta_1 & \sin\frac{n\beta_1}{4n-1} & \sin 3\frac{n\beta_1}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_1}{4n-1} \\ \sin n\beta_2 & \sin\frac{n\beta_2}{4n-1} & \sin 3\frac{n\beta_2}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_2}{4n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ \sin n\beta_{2n-1} & \sin\frac{n\beta_{2n-1}}{4n-1} & \sin 3\frac{n\beta_{2n-1}}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_{2n-1}}{4n-1} \end{vmatrix} \tag{4-18}$$

$$\Delta_{2n-1} = \begin{vmatrix} \sin\frac{n\beta_1}{4n-1} & \sin 3\frac{n\beta_1}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_1}{4n-1} \\ \sin\frac{n\beta_2}{4n-1} & \sin 3\frac{n\beta_2}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_2}{4n-1} \\ \vdots & \vdots & & \vdots \\ \sin\frac{n\beta_{2n-1}}{4n-1} & \sin 3\frac{n\beta_{2n-1}}{4n-1} & \cdots\cdots & \sin(4n-3)\frac{n\beta_{2n-1}}{4n-1} \end{vmatrix} \tag{4-19}$$

In the above determinants the β_j's are the (2n-1) inadmissible roots of equation (3-23), and these are given by

$$\begin{aligned} \beta_j &= \theta_1+\pi+\frac{\pi}{n}(n-j) & j &= 1,\ldots\ldots n \\ \beta_{j+s} &= -\theta_1+\frac{\pi}{n}(n-s) & s &= 1,\ldots\ldots n-1 \end{aligned} \tag{4-20}$$

By using a procedure similar to that used in Section III we have at once that

$$\begin{aligned} (u)_{ff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})} & \frac{\left(\frac{\tilde{c}_n}{k}\right)\sin\frac{n\theta}{4n-1}}{\prod\limits_{p=1}^{n}\left[\sin\left(\frac{\pi p}{n}+\theta\right)-\sin\theta_1\right]} \\ \cdot & \left[+\frac{\sin n\theta}{\sin\frac{n\theta}{4n-1}} - \sum_{j=1}^{2n-1}\frac{\sin n\beta_j}{\sin\frac{n\beta_j}{4n-1}}\; \frac{\prod\limits_{\substack{p=1\\(p\neq j)}}^{2n-1}\sin\frac{n(\beta_p-\theta)}{4n-1}\sin\frac{n(\beta_p+\theta)}{4n-1}}{\prod\limits_{p=1}^{2n-1}\sin\frac{n(\beta_p-\beta_j)}{4n-1}\sin\frac{n(\beta_p+\beta_j)}{4n-1}}\right] \end{aligned} \tag{4-21}$$

where

$$\tilde{c}_n = -\pi k\frac{1}{2^{n-1}} \tag{4-22}$$

The above constant $\tilde{c}_n$ can be found by considering the limiting case of $\theta_1 = 0$ $(\lambda = 0)$ where the solution is $i\pi H_0^{(1)}(kr)$ independently of n.* This result is the diffracted far field due to a line source located at the tip of a wedge of angle $\frac{\pi}{2n}$ (exterior angle $2\pi - \frac{\pi}{2n}$) when an impedance condition is prescribed on the upper face and a normal derivative boundary condition is prescribed on the lower. (It is also the result for the diffracted far field for a wedge of exterior angle $(4\pi - \frac{\pi}{n})$ when the same impedance boundary condition is prescribed on both surfaces).

V. Applications

1. *Right Angled Wedge Excited by a Line Source Located at the Tip With an Absorbing Upper Face and a Perfectly Conducting Lower Face.* For a right angled wedge n = 1. For an absorber the appropriate root is $\beta_1 = \theta_1 + \pi$ where $0 \leq \theta_1 \leq \pi$. Using equation (4-21) we find that the desired radiated far field is given by

$$(u)_{ff} = \sqrt{\frac{2}{\pi kr}}\; e^{i(kr+\frac{\pi}{4})} \frac{\left(\frac{\tilde{c}_1}{k}\right)\sin\frac{\theta}{3}}{[\sin(\pi+\theta)-\sin\theta_1]} \left[\frac{\sin\theta}{\sin\frac{\theta}{3}} - \frac{\sin\beta_1}{\sin\frac{\beta_1}{3}}\right] \tag{5-1}$$

where $\tilde{c}_1 = -\pi k$.

---

*The calculation involves noting that

$$i\pi H_0^{(1)}(kr) \to \sqrt{\frac{2}{\pi kr}}\; e^{i(kr+\frac{\pi}{4})}$$

and

$$\prod_{p=1}^{n} \sin\left(\frac{\pi p}{n}+\theta\right) = -\frac{1}{2^{n-1}}\sin n\theta$$

2. Forty Five Degree Wedge Excited by a Line Source Located at the Tip With an Absorbing Upper Face and a Perfectly Conducting Lower Face. For a forty five degree wedge n = 2. For an absorber the appropriate roots are $\beta_1 = \theta_1 + \pi$, $\beta_2 = \frac{\pi}{2} - \theta_1$ and $\beta_3 = \frac{3\pi}{2} + \theta_1$. Using equation (4-21) we find that the desired radiated far field is given by

$$(u)_{ff} = \sqrt{\frac{2}{\pi k r}}\, e^{i(kr+\frac{\pi}{4})} \frac{\left(\frac{\tilde{c}_2}{k}\right)\sin\frac{2\theta}{7}}{\left[\sin\left(\frac{\pi}{2}+\theta\right)-\sin\theta_1\right]\left[\sin(\pi+\theta)-\sin\theta_1\right]}$$

$$\cdot\left[\frac{\sin 2\theta}{\sin\frac{2\theta}{7}} - \frac{\sin 2\beta_1}{\sin\frac{2\beta_1}{7}}\,\frac{\sin\frac{2}{7}(\beta_2-\theta)\sin\frac{2}{7}(\beta_2+\theta)\sin\frac{2}{7}(\beta_3-\theta)\sin\frac{2}{7}(\beta_3+\theta)}{\sin\frac{2}{7}(\beta_2-\beta_1)\sin\frac{2}{7}(\beta_2+\beta_1)\sin\frac{2}{7}(\beta_3-\beta_1)\sin\frac{2}{7}(\beta_3+\beta_1)}\right. \tag{5-2}$$

$$- \frac{\sin 2\beta_2}{\sin\frac{2\beta_3}{7}}\,\frac{\sin\frac{2}{7}(\beta_1-\theta)\sin\frac{2}{7}(\beta_1+\theta)\sin\frac{2}{7}(\beta_3-\theta)\sin\frac{2}{7}(\beta_3+\theta)}{\sin\frac{2}{7}(\beta_1-\beta_2)\sin\frac{2}{7}(\beta_1+\beta_2)\sin\frac{2}{7}(\beta_3-\beta_2)\sin\frac{2}{7}(\beta_3+\beta_2)}$$

$$\left. - \frac{\sin 2\beta_3}{\sin\frac{2\beta_3}{7}}\,\frac{\sin\frac{2}{7}(\beta_1-\theta)\sin\frac{2}{7}(\beta_1+\theta)\sin\frac{2}{7}(\beta_2-\theta)\sin\frac{2}{7}(\beta_2+\theta)}{\sin\frac{2}{7}(\beta_1-\beta_3)\sin\frac{2}{7}(\beta_1+\beta_3)\sin\frac{2}{7}(\beta_2-\beta_3)\sin\frac{2}{7}(\beta_2+\beta_3)}\right]$$

where $\tilde{c}_2 = -\frac{k}{2}\pi$.

VI. Application of Method to Simple Boundary Conditions as in the Case of Perfect Conductors

The method employed above can also be employed with modification in treating the more ordinary classical diffraction problem, arising in the case of perfect conductors. Here we illustrate the method in the case of a half plane and a right angled wedge.

1. Diffraction by a Perfectly Conducting Half Plane

Consider the problem described below:

$$u_{xx} + u_{yy} + k^2 u = 0 \quad , \quad 0 \le \theta \le 2\pi \quad . \tag{6-1}$$

$$u = e^{ik(x\cos\theta_0 + y\sin\theta_0)} + \text{OUTGOING WAVE} . \tag{6-2}$$

$$u(-\infty, y) = e^{ik(x\cos\theta_0 + y\sin\theta_0)} , \; -\frac{\pi}{2} < \theta_0 < +\frac{\pi}{2} \quad . \tag{6-2a}$$

$$u(x,0) = 0 \quad , \; x > 0 . \tag{6-3}$$

$$u \text{ FINITE AT } x=0, y=0 \quad . \tag{6-4}$$

Here* we let $-\pi < \theta_0 < \pi$.

Let

$$v = \frac{\partial u}{\partial x} - ik\cos\theta_0 \; u \tag{6-5}$$

Then v is an outgoing wave function, which obeys the radiation condition. Also, from (6-3) and (6-5) we have that $v(x,0) = 0$, $x > 0$. Therefore (6-3) implies that the most general form of v is given by

$$v = \sum_{n=1}^{\infty} a_n H^{(1)}_{\frac{n}{2}}(kr) \sin\frac{n}{2}\theta \tag{6-6}$$

where the a_n are to be determined. Now (6-5) shows that $v = O(\frac{u}{x})$ near the origin; therefore (6-4) implies that $u \to 0$ at origin, and also that $a_n = 0$, $n > 1$. Therefore

$$v = a_1(\theta_0) H^{(1)}_{\frac{1}{2}}(kr) \sin\frac{\theta}{2} = \frac{\partial u}{\partial x} - ik\cos\theta_0 \; u \tag{6.6a}$$

We have yet to determine a_1.

Now let

$$u = u_{geom} + u^{(1)} \tag{6.7}$$

---

*The definition of the angle θ_0 employed in this section differs from that used in earlier sections. Henceforth θ_0 means the angle between the direction of travel of the plane wave and the positive x-axis.

and assume

$$u^{(1)} \to \tilde{H}_0^{(1)}(kr)\,u(\theta) \quad , \qquad r \to \infty \tag{6-8}$$

where

$$\tilde{H}_0^{(1)}(kr) \equiv \sqrt{\frac{2}{\pi k r}}\; e^{ikr - i\frac{\pi}{4}} \tag{6-9}$$

Equation (6-8) holds for almost all fixed θ. We also know that (6-5) holds, and that

$$v \to \tilde{H}_0^{(1)}(kr)\, e^{-i\frac{\pi}{4}} a_1 \sin\frac{\theta}{2} + o\left(\frac{1}{\sqrt{r}}\right) \quad . \tag{6-10}$$

But

$$\frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial r} - \frac{1}{r}\sin\theta \frac{\partial}{\partial \theta} \tag{6-11}$$

while for large r,

$$\begin{aligned} &\frac{\partial}{\partial r}\tilde{H}_0^{(1)}(kr) \sim ik\,\tilde{H}_0^{(1)}(kr) + o\left(\frac{1}{\sqrt{r}}\right) \\ &\frac{1}{r}\tilde{H}_0^{(1)}(kr) \sim O\left(\frac{1}{r^{3/2}}\right) \end{aligned} \tag{6-12}$$

Therefore, for functions of the type illustrated in (6-8) and (6-10), $\frac{\partial}{\partial x} \longrightarrow ik\cos\theta$. Using (6-5) and (6-10) the coefficient of $H_0^{(1)}$ in (6-10) now becomes

$$a_1 e^{-i\frac{\pi}{4}} \sin\frac{\theta}{2} = ik(\cos\theta - \cos\theta_0)\,u(\theta) .$$

Hence,

$$u(\theta) = \frac{a_1 \sin\frac{\theta}{2}\; e^{-i\frac{\pi}{4}}}{ik(\cos\theta - \cos\theta_0)} \tag{6-13}$$

where, according to (6-8), u(θ) is the (physically interesting) complex far field amplitude of the diffracted wave. The θ dependence is quite explicit.

We need the constant a_1, which may depend only on θ_o. To emphasize the dependence on the angle of incidence we rewrite (6-8) as follows:

$$u(\theta,\theta_0) \equiv u(\theta) = \frac{a_1(\theta_0)\sin\frac{\theta}{2}\,e^{-i\frac{\pi}{4}}}{ik(\cos\theta-\cos\theta_0)} \tag{6-14}$$

Now by reciprocity

$$u(\theta,\theta_0) = u\,(\theta_0+\pi,\theta-\pi) \tag{6-15}$$

Here we have employed the convention that angles of incidence lie in the range $(-\pi, \pi)$. Applying (6-15) to Eqn. (6-14) we find

$$\frac{a_1(\theta_0)\sin\frac{\theta}{2}}{ik(\cos\theta-\cos\theta_0)} = \frac{a_1(\theta-\pi)\sin\frac{\theta_0+\pi}{2}}{ik[\cos(\theta_0+\pi)-\cos(\theta-\pi)]} = \frac{a_1(\theta-\pi)\sin\frac{\theta_0+\pi}{2}}{ik(\cos\theta-\cos\theta_0)} \tag{6-16}$$

i.e.

$$\frac{a_1(\theta_0)}{\sin\frac{\theta_0+\pi}{2}} = \frac{a_1(\theta-\pi)}{\sin\frac{\theta}{2}} = \text{CONSTANT} \tag{6-17}$$

or

$$a_1(\theta_0) = a_1(0)\cos\frac{\theta_0}{2} \tag{6-18}$$

$$u(\theta,\theta_0) = \left(-ike^{-i\frac{\pi}{4}}\right)\frac{\cos\frac{\theta_0}{2}\sin\frac{\theta}{2}\,e^{-i\frac{\pi}{4}}}{ik(\cos\theta-\cos\theta_0)} \;. \tag{6-19}$$

Here we have used (6-18) with a value of $a_1(0)$ obtained in the next paragraph.

In order to determine $a_1(0)$ we let $\theta_o = 0$, and employ (6-6a) and (6-5). We integrate (6-5), subject to (6-2a), and find

$$u = e^{ikx} + a_1(0)\, e^{ikx} \int_{-\infty}^{x} e^{-ik\xi} \chi(\xi,y)\, d\xi \tag{6-20}$$

where

$$\chi(x,y) = H^{(1)}_{\frac{1}{2}}(kr) \sin\frac{\theta}{2} \rightarrow -i\sqrt{\frac{2}{\pi k r}}\, e^{ikr} \sin\frac{\theta}{2} \tag{6-21}$$

$$\begin{aligned} &\chi(x,0) = 0 \\ &\chi(x,0) = -i\sqrt{\frac{2}{\pi k}}\, \frac{e^{+ik|x|}}{\sqrt{|x|}} \qquad , \; x < 0 . \end{aligned} \tag{6-21a}$$

Imposing the boundary condition (6-3), and using (6-21a) we get

$$0 = e^{ikx} + a_1(0)\, e^{ikx} \int_{|\xi|=\infty}^{0} (-i)\sqrt{\frac{2}{\pi}}\, \frac{e^{2ik|\xi|}}{\sqrt{k|\xi|}}\, \frac{d(-k|\xi|)}{k} \tag{6-21b}$$

Hence, finally,

$$a_1(0) = \left[\frac{i}{k\sqrt{\pi}} \int_0^{\infty} \frac{e^{it}}{\sqrt{t}}\, dt \right]^{-1} = -ik e^{-i\frac{\pi}{4}} \tag{6-22}$$

so that

$$u(\theta,\theta_0) = \frac{i \cos\frac{\theta_0}{2} \sin\frac{\theta}{2}}{(\cos\theta - \cos\theta_0)} . \tag{6-22a}$$

The result is that for diffraction by a half plane under the condition (6-3), the field for large r is given by

$$u \rightarrow u_{geom} + \sqrt{\frac{2}{\pi k r}}\, e^{-i\frac{\pi}{4}} e^{ikr}\, \frac{i\cos\frac{\theta_0}{2}\sin\frac{\theta}{2}}{(\cos\theta - \cos\theta_0)} . \tag{6-23}$$

Similar analysis shows that when $\frac{\partial u}{\partial y} = 0$ is required at the screen has the form

$$u \to u_{geom} + \sqrt{\frac{2}{\pi k r}}\, e^{-i\frac{\pi}{4}} e^{ikr} (\text{CONSTANT}) \frac{\sin\frac{\theta_0}{2}\cos\frac{\theta}{2}}{ik(\cos\theta - \cos\theta_0)} \cdot \qquad (6\text{-}24)$$

The analysis given above through equation (6-6a) can be traced back to Lamb[7], in the case of normal incidence ($\theta_0 = \frac{\pi}{2}$), except Lamb omits equation (6-6) and merely postulates (6-6a). No explicit formulation of the edge condition was available to him. After obtaining (6-6a) Lamb expresses it in the form (again for $\theta_0 = \frac{\pi}{2}$)

$$\cos\theta \frac{\partial u}{\partial r} - \frac{1}{r}\sin\theta \frac{\partial u}{\partial \theta} = (\text{CONSTANT}) \frac{e^{ikr}}{\sqrt{r}} \sin\frac{\theta}{2}$$

However, Lamb integrates the 1st order partial differential equation to obtain the usual Fresnel integral expression for u. The form of the resulting solution is then generalized to be valid for arbitrary θ_0 .

2. Diffraction by a Perfectly Conducting Wedge

The present procedure can also be employed in the case of a perfectly conducting wedge.

For example, consider the problem of diffraction by a perfectly conducting right angled wedge.

$$u_{xx} + u_{yy} + k^2 u = 0 \qquad (6.25)$$

$$u = e^{ik(x\cos\theta_0 + y\sin\theta_0)} + \text{OUTGOING WAVES} \qquad (6.26)$$

$$u = u_{geom} \text{ AT INFINITY} \qquad (6.26a)$$

$$u(x,0) = 0 \quad , x > 0 \qquad (6.27)$$

$$u(0,y) = 0 \quad , y < 0 \qquad (6.27a)$$

$$u \text{ FINITE AT } x=0, y=0 \tag{6-28}$$

Let

$$v = \left(\frac{\partial^2}{\partial x^2} + k^2 \cos^2\theta_0\right) u = -\left(\frac{\partial^2}{\partial y^2} + k^2 \sin^2\theta_0\right) u \tag{6-28a}$$

Then the geometric or plane wave part of v is zero. Again, v = 0 on the faces of the wedge, and v is not too singular to prevent u from being finite . Hence

$$v = a_1(\theta_0) H^{(1)}_{\frac{2}{3}}(kr) \sin\frac{2\theta}{3} + a_2(\theta_0) H^{(1)}_{\frac{4}{3}}(kr) \sin\frac{4\theta}{3} \quad , 0 < \theta < \frac{3\pi}{2} \tag{6-29}$$

so that

$$v \to \tilde{H}^{(1)}_0(kr) \left[e^{-i\frac{\pi}{3}} a_1(\theta_0) \sin\frac{2\theta}{3} + e^{-i\frac{2\pi}{3}} a_2(\theta_0) \sin\frac{4\theta}{3} \right] \tag{6-30}$$

Therefore,

$$u(\theta, \theta_0) = \frac{e^{-i\frac{\pi}{3}} a_1(\theta_0) \sin\frac{2\theta}{3} + e^{-i\frac{2\pi}{3}} a_2(\theta_0) \sin\frac{4\theta}{3}}{-k^2(\cos^2\theta - \cos^2\theta_0)} \tag{6-31}$$

Now let

$$a_1 e^{-i\frac{\pi}{3}} = \alpha_1 \quad , \quad a_2 e^{-i\frac{2\pi}{3}} = \alpha_2 \tag{6-32}$$

Hence

$$\alpha_1(\theta_0) \sin\frac{2\theta}{3} + \alpha_2(\theta_0) \sin\frac{4\theta}{3} = -\Big[\alpha_1(\theta-\pi) \sin\tfrac{2}{3}(\theta_0+\pi) + \alpha_2(\theta-\pi) \sin\tfrac{4}{3}(\theta_0+\pi) \Big] \tag{6-33}$$

since $u(\theta, \theta_0) = u(\theta_0+\pi, \theta-\pi)$ because of reciprocity.

Here let $\theta = \theta_0 + \pi$, and deduce

$$\alpha_1(\theta_0)\sin\tfrac{2}{3}(\theta_0+\pi) + \alpha_2(\theta_0)\sin\tfrac{4}{3}(\theta_0+\pi) = 0$$

or

$$\alpha_2(\theta_0) = \frac{-\alpha_1(\theta_0)}{2\cos\frac{2}{3}(\theta_0+\pi)} \tag{6-34}$$

Thus, substituting in (6-33),

$$\sin\tfrac{2\theta}{3}\,\alpha_1(\theta_0)\left[1 - \frac{\cos\frac{2\theta}{3}}{\cos\frac{2}{3}(\theta_0+\pi)}\right] = -\alpha_1(\theta-\pi)\sin\tfrac{2}{3}(\theta_0+\pi)\left[1 - \frac{\cos\frac{2}{3}(\theta_0+\pi)}{\cos\frac{2}{3}\theta}\right] \tag{6-35}$$

Let $\theta = \pi$, and obtain

$$\alpha_1(\theta_0) = \frac{-\alpha_1(0)\sin\frac{2}{3}(\theta_0+\pi)\left[1+2\cos\frac{2}{3}(\theta_0+\pi)\right]}{\sin\frac{2\pi}{3}\left[1+\frac{1}{2\cos\frac{2}{3}(\theta_0+\pi)}\right]} = \frac{-2\alpha_1(0)}{\sqrt{3}}\sin\tfrac{4}{3}(\theta_0+\pi) \tag{6-36}$$

Hence

$$u(\theta,\theta_0) = \frac{-\frac{2}{\sqrt{3}}\alpha_1(0)\sin\frac{4}{3}(\theta_0+\pi)}{k^2(\cos^2\theta_0-\cos^2\theta)}\left[\sin\tfrac{2\theta}{3} - \frac{2\sin\frac{2\theta}{3}\cos\frac{2\theta}{3}}{2\cos\frac{2}{3}(\theta_0+\pi)}\right] \tag{6-37}$$

or

$$u(\theta,\theta_0) = -\frac{4\alpha_1(0)}{\sqrt{3}}\,\frac{\sin\frac{2}{3}(\theta_0+\pi)\sin\frac{2}{3}\theta\left[\cos\frac{2}{3}(\theta_0+\pi)-\cos\frac{2}{3}\theta\right]}{k^2(\cos^2\theta_0-\cos^2\theta)} \tag{6-38}$$

The constant $\alpha_1(0)/k^2$ is a pure number as the form of (6-38) shows. This number can be obtained by using the same sort of procedure as we used in the case of a half plane.

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